Verification and Renewal Curriculum

Euclidean Geometry: Instructor's Syllabus

A VRC Curriculum Syllabus

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Introduction & Purpose

Geometry in public schools today has lost its purpose. Many, including students, parents, and teachers alike, question the importance of such a subject in a student's core education. It is often considered an application of algebra to geometric principles with a small portion of time spent on the study of proofs and little consideration for geometry itself. The use of proofs is the foundational system of geometric learning conceived by the ancients, and it is through repetition of such proofs through reading, memorization, presentation, and discussion that a student gains the faculties of geometrical imagination, logical deduction, and reasoning that is at the core of a good education.

The aim of the Euclidean Geometry class is to develop a student's logical reasoning through discovering and proving the basic tenets of geometry. By proving the propositions present in Euclid's *The Elements*, students gain a deeper understanding of the principles that underlie geometric reasoning. Such an approach to the subject teaches students geometric theorems and their origins through demonstrative proofs and further develops and concretizes essential skills in logical deduction and reasoning.

Course Objectives

1) The ability to read and employ definitions, postulates, and common notions in geometric reasoning.
2) To memorize and present deductive argumentation, i.e., geometric propositions with clarity and understanding.
3) To visualize and demonstrate the manipulation of geometric figures through second- and third-dimensional space.
4) To discuss the relationships between geometric propositions.
5) To discover and assess the underlying principles guiding geometric figures.
6) To learn how to hold a respectful, intellectual conversation with peers.
7) To be able to listen well.

Potential Texts

Primary Texts/Readings

- *The Elements* by Euclid translated by Sir Thomas Heath (Students should read the version with only the propositions without commentary, while teachers and parents should read Heath's commentary.)

Secondary Texts/Readings

- *Euclid's Elements Book One with Questions for Discussion* by Dana Densmore (essential for teachers teaching Euclid for the first time)
- *The First Six Books of The Elements of Euclid in which Coloured Diagrams and Symbols are Used Instead of Letters for the Greater Ease of Learners* by Oliver Byrne (Another way of demonstrating the propositions for those students that prefer visual aids.)
- *Sacred Geometry* (Wooden Books)
- *Spiritual Significance in Islamic Architecture* by Mostafa Badawi

Recommended Modern Geometry Textbooks

- *Geometry: A High School Course Second Edition* by Serge Lang and Gene Murrow
- *Geometry, Grades 9-12: McDougal Littel High School Math*
- *Introduction to Geometry* by Richard Rusczyk
- *Kiselev's Geometry Book 1 and 2* by A.P. Kiselev

Teacher Resources

- Euclid Videos: Sandy Bultena's *Euclid's Elements Book 1* playlist[1]

Student Materials

- A notebook with geometric paper (preferred), or normal lined paper.
- A geometric compass.
- A ruler.
- A whiteboard or chalkboard (even sidewalk chalk for fun).

Different Approaches to Class Structure

Depending on time and aims, there are three different approaches to undergoing a Euclidean geometry course that incorporates Euclid's *Elements*.

The first approach is to simply have the students work through all of Euclid's propositions. This requires the most amount of time and may be too tedious for the average student; however, for those students who enjoy Euclid, this will be the most fruitful option.

The second approach is to work through the condensed list of propositions selected from Euclid's *Elements*. This can also be thematically supplemented with a modern geometry textbook. *Geometry: A High School Course* may be the best option for this approach.

The third approach is to go through Book I of Euclid's *Elements* with the students and then switch to a modern textbook on geometry. While working through the modern textbook,

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[1] "Euclid's Elements Book 1," posted February 19, 2014, by Sandy Bultena, YouTube, https://www.youtube.com/watch?v=ewir8jyohyc&list=PLrkQ3hzZrc4j9gToz--_CiFzQLeVb32hQ.

the students should continue to go through Books II-VI at a slower pace to continue to practice and refine their demonstration skills. This has the benefit of allowing students to practice pure proofs and gain a sense for geometry before they encounter it in its modern form while still practicing the skill of deductive reasoning.

Condensed List of Euclid's Propositions[2]

Book I - Basic of Plane Geometry Focusing on Triangles

1. Elements, Book I, Definitions, Postulates, Common Notions (have students memorize these)
2. Propositions 1-2
3. Props. 3-5[3]
4. Props. 6-8
5. Props. 9-12
6. Props. 13-16
7. Props. 17-21
8. Props. 22-25
9. Props. 26-29
10. Props. 30-34
11. Props. 35, 36*-41*, 42-46
12. Props. 47-48
13. Discussion of Bk I

Book II - Geometric Algebra

14. Definitions, Props. 1-4
15. 5-7, 8*, 9, 10*
16. Props. 11-14

Book III - Circles and their Properties

17. Definitions, Props. 1-3, 4*, 6*
18. Props. 7-8, 9*-13*
19. Props. 14-16,17*-19*
20. Props. 20-21, 22*-23*
21. Props. 24*-30*, 31-33
22. Props. 34*, 35-37

Book IV - Incircle and Circumcircle of Triangles; Polygons

23. Definitions, Props. 1, 2, 3*, 4*, 5, 6*, 7*, 8*, 9*
24. Props. 10, 11, 15 (read 12*-14*, 16*)

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[2] * Indicates that the proposition should be read and discussed only.

[3] Proposition 5 is notoriously hard for the beginning of geometry. Tell the students that it indeed gets easier after it.

Book V - Proportions of Magnitudes

25. Definitions 1-2, props. 1-3
26. Definitions 3-6, prop 4
27. Definitions 5 and 13, Props. 5-7
28. Definitions 7-8, props 8-11
29. def 11, props 12-15
30. Definitions 12-15, Props. 16-18
31. Definitions 16-18, Props. 19-22, read Props. 23*-25*
32. Review

Book VI - Application of Proportions to Plane Geometry

33. Definitions, Props. 1-3
34. Props. 4, 5, 6*, 7*, 8
35. Props. 10-14
36. Props. 16, 17, 19, 23, 24*-26*
37. Props. 27*-29*, 30, 31*-33*

Book VII - Number Theory

38. definitions, props 1, 2, 3*, 4
39. Props. 5*-15*, with number examples
40. Props. 19, 20*, 21
41. Props. 22, 23*, 24, 25*-30*, 31
42. Review

Book VIII - Geometric Sequences of Integers

43. Props. 1,5,11; read 2*-4* and 10*
44. Props. read 12*, 18*, 22*-27*

Book IX - Application of Previous Two Books; Infinitude of Prime Numbers; Construction of Even Perfect Numbers

45. 16, 18, 20; read 35*-36*

Book X - Irrationality of the Square Roots of Non-Square Numbers; Square Roots of Incommensurable Lines; Pythagorean Triples

46. definitions, and props 1,2,5; read Props. 3*-4*
47. Props. 7, 12; read 6*

Book XI - Results of Book VI applied to Solids

48. definitions and prop 1
49. Props. 2-4
50. Props. 5-7; read 8*
51. Props. 9-12
52. Props. 13-15
53. Props. 20-21; read 16*-19*
54. Props. 24, 27-29; read 25*

55. Props. 33, 38; read 30*-32*

Book XII - Volumes of Cones, Pyramids, and Cylinders

56. Props. 1,2
57. Props. 7*, 10
58. Props. 18 and review

Book XIII - Inscribing Five Platonic Solids in a Sphere

59. Props. 1-4
60. Props. 5, 6*, 7, 8
61. Props. 9, 10, 12
62. Props. 13-15, 17*
63. Props. 18

Teacher Guidance

The role of the instructor is crucial to this course because he must guide the student, especially when the student has lost his way in a proposition. The instructor, like the student, should have prepared and memorized the propositions to be presented for the day's lesson. Using leading questions, he must be able to gently guide the student out of any difficulties. The teacher should never just fix an error in the student's presentation or show the student what he did wrong until he has exhausted all possible hints. He should help the student help himself out of the difficulty (see the ***How a Proposition should be Presented*** section below). This is pivotal in supporting and developing the students' reasoning faculty.

It is important that teachers, or parents (in the case of homeschoolers), read Heath's commentary while going through the propositions with students. This can be a source of enrichment for students since it includes different ways to solve the propositions, algebraic equivalents, and references to modern geometry—it is an essential tool.

It is also advised that teachers and parents read ahead in the text in order to conceptualize the aim of the overall book they are working through. Euclid arranged his text for specific reasons. It is important for students and teachers to try to discover these. Once a student completes the propositions in a specific book, these questions can be asked in a discussion:

What is the theme of this book?

Why did Euclid put these propositions in this order?

What is the relation between these propositions and those in previous books?

What is the "story" of this book?

Why did Euclid structure his entire book this way? (Asked at the end)

Finally, Book XIII of Euclid is the conclusion to his story. In it he inscribes all of the platonic solids into spheres. These solids, according to Plato, were the building blocks, the "elements," of the cosmos.

Muṭālaʿa (Pre-reading)

This portion of the class requires students to prepare their propositions. They do this by reading the proposition, understanding them, and memorizing the steps to the proposition. A student's presentation requires him to go through the entire proposition step-by-step, drawing out each step to make a diagram of the proposition. The student should utilize a ruler and compass to draw his propositions. He should also practice the proposition several times from memory before presenting.

It should be noted that there is a learning curve that comes with a student's ability to prepare his propositions independently. Students should receive ample guidance and practice at the beginning of the course (class-time, office hours, etc.) to ensure they understand what is required of them. Support at home from parents is also helpful to students at this stage. Over the course of the class students should be able to independently read and prepare propositions, and the class expectation should shift accordingly.

Mudhākara (Group Review)

This portion of the class takes place outside of class hours. It should be assigned for homework. Ideally, a separate time for *mudhākara* would be set up by the school or parents to ensure students have ample time to complete it together. Students get together in assigned groups and review the propositions completed in class that week by reading through them and presenting them again together. This is a space for students to work with their fellow peers to seek clarity, ask questions, and engage in deeper conversation around the text.

Classwork and Homework Ideas

The classwork required for the course is encapsulated in the daily presentations students must present to their peers and instructors. This could incorporate a grade that evaluates the presentation. The presentation grade would incorporate factors like completeness, clarity, following steps, and more. Please see a sample scale below.

Euclid Proposition Grading Scale	
Prepared (*muṭālaʿa*)	40%
Completed proposition	20%
Explained all steps w/references	20%
Clarity in presentation	20%

For students who are not presenting, classwork incorporates participation through asking the presenter questions, seeking clarity where there is confusion, and more. Furthermore, it is important that every student has their own simplified version of the presentations in their notebooks. This could be incorporated into the students' homework.

Students would be required to prepare for their presentations at home. This would include that each student, regardless of whether they are presenting, must have each proposition written out in a simplified notation to aid their memorization of the proposition. It is also important that students label their propositions differently from the letters that appear in Euclid's text. This way students' minds can begin to get accustomed to abstracting the logical steps of each proposition. This is essential to helping students acquire the logical act of applying universals to particulars. In such cases, students apply the steps of a given proposition to the particular labels they have chosen for their individual diagrams. Students can make their own propositions in English or another language, if preferred, as long as the explanation of the proposition remains clear.

Another idea for homework is to ask students to present their propositions to siblings and other family members. This forces them to be clear since their other family members may have less experience in geometry than their teachers.

Quizzes and Tests

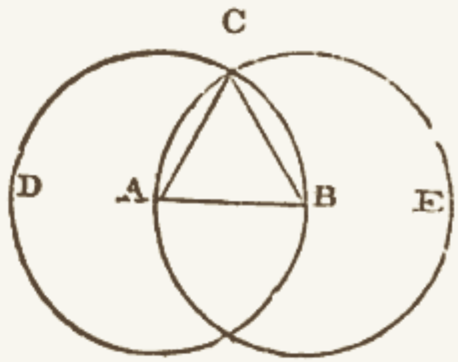
Every proposition presentation is a mini quiz. Still, it often benefits students to have them quizzed on the definitions, postulates, and common notions/axioms to aid their memorization. Teachers can also ask students to memorize the enunciations of propositions and quiz them on them. Another option is to have students write out a proposition or several for a test.

How a Proposition should be Presented

PROPOSITION I. PROBLEM.

To describe an equilateral triangle upon a given finite straight line.

Let AB be the given straight line; it is required to describe an equilateral triangle upon it.



From the centre A, at the distance AB, describe (3. Postulate.) the circle BCD, and from the centre B, at the distance BA, describe the circle ACE; and from the point C, in which the circles cut one another, draw the straight lines (2. Post.) CA, CB to the points A, B; ABC shall be an equilateral triangle.

Because the point A is the centre of the circle BCD, AC is equal (15. Definition.) to AB; and because the point B is the centre of the circle ACE, BC is equal to BA: but it has been proved that CA is equal to AB; therefore CA, CB are each of them equal to AB; but things which are equal to the same are equal to one another; (1st. Axiom.) therefore CA is equal to CB; wherefore CA, AB, BC are equal to one another; and the triangle ABC is therefore equilateral, and it is described upon the given straight line AB. Which was required to be done.

The image above is of Euclid's first proposition in Book I. If a student were presenting this proposition, they would first begin by stating the enunciation[4] as if the proposition were theirs, not Euclid's:

Speaker	**Action**
Student: "Today I will be presenting Proposition 1. I will be constructing an equilateral triangle upon a given straight line (enunciation, which is universal)."	The student then tells and draws for us his "givens" and his "to prove" or "to construct" statements.

[4] The enunciation is usually the first line of the proposition. In some copies of The Elements it is in italics. It encapsulates the entire proposition in a single sentence in a universal way rather than particular (to create an equilateral triangle from a given finite line, not to create an equilateral triangle ABC on the given line AB). Students should have enunciations memorized because it facilitates the demonstrations of the propositions.

Student: “I’m given the finite straight line AB. I intend to construct on the given line AB an equilateral triangle (to construct, which is particular).”	The student writes “Given: AB,” and “To Construct: equilateral triangle on AB,” then he begins the “steps” of the proposition. He should draw the images equivalent to the steps on the board and number each step (this is crucial for when propositions become long). At each step the student must give his reason with a reference[5] in brackets.
Student: “From center A with distance AB, I construct a circle BCD, which I know I can do because of Euclid’s postulate 3 that states I can describe a circle with any center, in this case A, and any distance, AB.”	The student then draws the circle and writes on the board “1. Circle BCD with distance AB [Postulate 3].”
Student: “Now, from center B, I construct the circle ACE with distance BA, which, again, I know I can do because of postulate 3.”	The student then draws the circle and writes on the board “2. Circle ACE with distance BA [Postulate 3].”
Student: “From A and B, I’m going to draw straight lines to the point where the circles meet, which I’ve labeled C.”	
Teacher: “How do you know you can do this?”	
Student: “Oh, yeah, I know I can do this because of Euclid’s second postulate which states that I can produce a finite straight line continuously in a straight line.”	The student then draws the two lines and writes on the board, “3. Line AC and BC [Postulate 2].”
Student: “I will now show you how this triangle ABC is equilateral.”	

At this point the student has finished the “construction” portion of the proposition. He must now prove that this is indeed an equilateral triangle. It is not enough to simply say, “I’ve done it.” Many students will struggle with the idea of “proving” something at first, thinking that it is tedious or a waste of time. Unfortunately, standardized education does not value this type of knowledge. It is however essential to the Islamic tradition.

Speaker	**Action**

[5] References can be definitions, common notions, postulates, or even previous propositions. Many copies of *The Elements* already provide a basic reference in brackets in the proposition itself.

Students: "AC is equal to AB because they are both straight lines from the center A of the circle BCD to the line that surrounds the circle (circumference). We know this because Euclid's definition of a circle is a figure contained by one line (circumference) such that all the straight lines falling upon it from one point (the center) among those lying within the figure are equal to one another (radii)."	The student then writes, "4. AC = AB [Definition 15]."
Teacher: "We call the line that surrounds the circle a 'circumference', and those lines that go from the center to the circumference are called 'radii'. A single one is called a 'radius'."[6]	
Student: "For the same reasons, we know that CB is equal to AB. That is, they are both radii of the circle ACE."	The student then writes "5. CB = BA [Definition 15]"
Student: "Okay, I'm stuck. I need a hint!"	
Teacher: "Do you see anything similar in steps 4 and 5?"	
Student: "No, I don't think so."	
Teacher: "Maybe you're confused because you wrote 'BA' in step 5. Isn't 'BA' the same as 'AB'?"	
Student: "Oh yes, so both CB and AC are equal to AB."	
Teacher: "And what do we know about things that are equal to the same thing?"	
Student: "Ah! We know from Euclid's first axiom/common notion that things which are equal to the same thing are equal to one another."	
Teacher: "Okay, and what things are equal to each other?"	The student then writes "AC = CB."

[6] It is fine for the teacher to provide some of the more technical vocabulary mentioned in modern textbooks for ease of reference.

Teacher: "Don't forget to number your steps and mention the axiom."	The student then writes "6. AC = CB [Axiom/Common Notion 1]."
Student: "So that means that AC is equal to CB and both are equal to AB, which means all the lines are equal to each other, which is what we were trying to prove."	
Teacher: "What do you mean 'That's what you were trying to construct?' What is your 'to construct' statement?"	
Student: "Right, the 'to construct' statement is 'To Construct: equilateral triangle on AB'." So, AC, CB, and AB are all equal to each other, and they are sides of a triangle, which we now know is an equilateral triangle, which is what we set out to construct. Q.E.F."	The student then writes " $\therefore$ AC = CB = AB and are sides of an equilateral triangle Q.E.F."

The three dots is the mathematical sign for "therefore." Q.E.F. stands for *Quod erat faciendum*, which means "which had to be done." It is used for constructions. Q.E.D. stands for *Quod erat demonstrandum*, which means "that which was to be demonstrated." It is used for demonstrations.

Above is a sample of a proposition demonstration with examples of some of the common struggles that students face when presenting. It also shows how a teacher or parent should aid the child in the demonstration. At the end of the demonstration, the whiteboard or chalkboard should look like this:

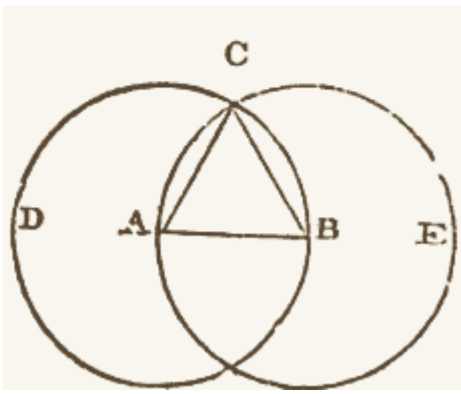
<table>
<tr><th colspan="2">Sample Proposition Demonstration</th></tr>
<tr><td colspan="2">Enunciation: To construct an equilateral triangle on a finite straight line.</td></tr>
<tr><td>Given: AB</td><td>To Construct: equilateral triangle on AB</td></tr>
<tr><td>
</td><td>Steps:
1. Circle BCD with distance AB [Post. 3]
2. Circle ACE with distance BA [Post. 3]
3. Line AC and BC [Post. 2]
4. AC = AB [Def. 15]
5. CB = BA [Def. 15]
6. AC = CB [C.N. 1]
7. ∴ AC = CB = AB and are sides of an equilateral triangle
Q.E.F</td></tr>
</table>

Figure 1[7]

Students should also draw little lines over pairs of letters to indicate that they are lines, and can draw little circles, triangles, and parallelograms rather than writing out “circle,” “triangle,” or “parallelogram.”

Note: Students should ***never*** draw the entire proposition and then talk through the proposition. Some students have a tendency to memorize the drawing and the steps but not put them together. It is essential to draw and explain each step one at a time as they draw the proposition so that their imagination can build the construction as their mind reasons through it. This is important in preventing students from getting confused or lost. It is even more essential for developing their imaginative faculties.

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[7] Students should label the propositions with “Book number in Roman numerals.” “Proposition number in Arabic numerals.”